Measuring Scales

To measure a length, a metre scale is generally used, which is graduated to centimeter and millimeter, and is one metre in length. For the measurement of a length with a metre scale we adopt the following procedure.

(a) Note the value of one smallest division of the scale.
(b) Hold the scale on its side such that marking of the scale are very close to the points between which the distance is to be measured.
(c) Take reading by keeping the eye perpendicular to the scale above the points for which measurement is made.
(d) Avoid using zero of the scale as it may be damaged. Measure the distance as a difference of two scale reading. For situations wherein, direct placing of the scale is inconvenient, use a divider. In this case the divider is set to the length to be measured and then transferred to the scale for actual measurement of the length.

The vernier or vernier scale is an additional scale. The vernier scale was invented in its modern form in 1631 by the French mathematician Pierre Vernier (1580–1637). In some languages, this device is called a **nonius**. It was also commonly called a nonius in English until the end of the 18^{th} century. *Nonius* is the Latin name of the Portuguese astronomer and mathematician Pedro Nunes (1502–1578) who in 1542 invented a related but different system for taking fine measurements on the astrolabe that was a precursor to the vernier.
A vernier scale slides across a fixed main scale. By using it a uniformly graduated main scale can be accurately read to a fractional part of a division. In principle, not only for measuring lengths, it can be used on any measuring device with a graduated scale. Verniers are common on sextants used in navigation, scientific instruments used to conduct experiments, machinists' measuring tools (all sorts, but especially calipers and micrometers) used to work materials to fine tolerances and on theodolites used in surveying.

<u>Types of Vernier</u>

Direct Vernier or Forward Vernier:
Refer to Fig. 1. In case of direct vernier both scales, namely vernier and main, move in the same direction and vernier divisions are marked in the same direction as that of the main scale.
As in Fig. 1(a) in direct vernier scale, n vernier divisions are compressed into the space of (n–1) main scale divisions, and we say the vernier-scale ratio is [(n) : (n – 1)].
Let d = value of smallest division on the main scale
v = value of smallest division on the vernier scale

Then, $v = \frac{n-1}{n} d$

The smallest distance measured using a vernier scale is known as the least count / vernier constant (LC).

LC = $d - v = d - \frac{n-1}{n} d = \frac{d}{n}$

Since the vernier scale shown in Fig 1(a). is constructed to have ten divisions in the space of nine on the main scale, any single division on the vernier scale is 0.1 divisions less than a division on the main scale. Naturally, this 0.1 difference can add up over many divisions.

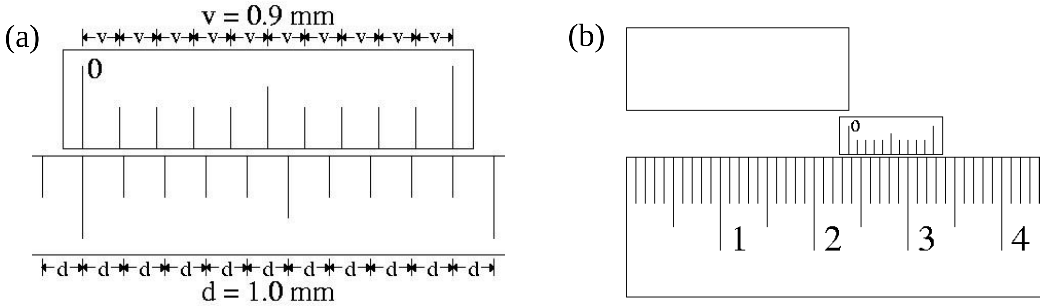

Figure 1. (a) Direct vernier (b) measuring the length of the rectangle using a direct vernier scale

So, the divisions on the vernier scale are not of a standard length, but the divisions on the main scale are always of some standard length e.g. millimeters. A vernier scale enables an unambiguous interpolation between the smallest divisions on the main scale.

Measurement using a vernier

For taking reading using a vernier scale note the division on the vernier scale that coincides with some division of the main scale. Multiply this number of vernier division with the vernier constant. This is the vernier scale reading. Record the observed reading by adding the vernier scale reading to the main scale reading. (Note: The main scale reading is the reading on the main scale which appears on or before the zero of the vernier scale.)

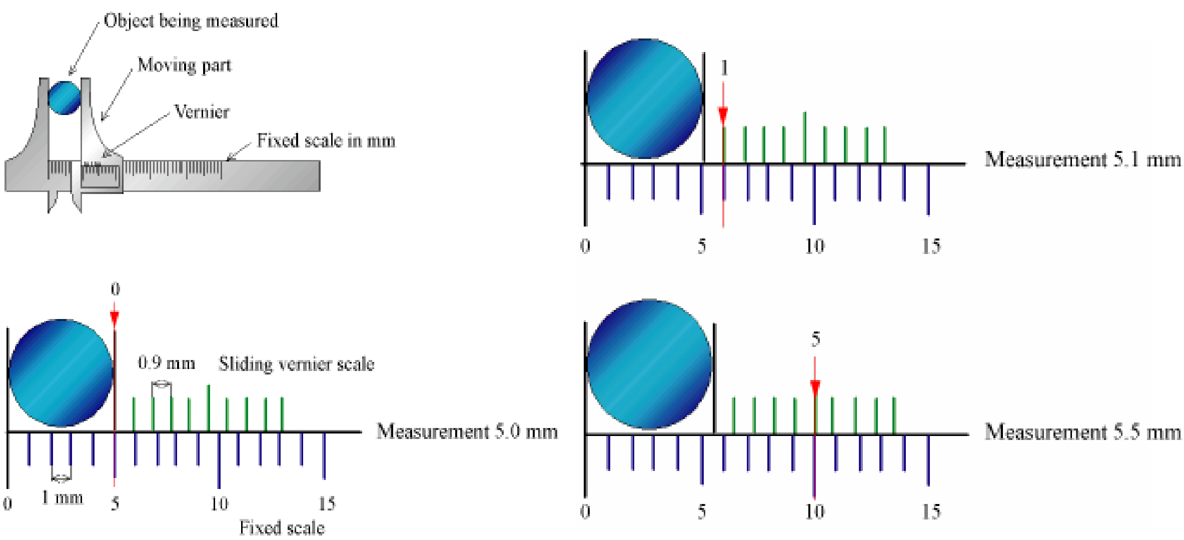

Figure 2. Measurement of diameter of circles of different radii using slide calipers, an instrument in which a vernier is attached.

Zero Error: When the two jaws are touching each other, the zero of the vernier scale may cross or lag behind the zero of the main scale. If *x* divisions of the vernier scale coincide with certain divisions on the main scale the instrument error is $y = x \times L.C.$ The error is *positive* when the vernier zero is on the right and *negative* when the vernier zero is on the left side of the main scale zero. Positive error is to be subtracted and negative error is to be added to the measured value.

Retrograde Vernier or Backward Vernier:
In case of retrograde vernier, vernier and main scales move in the opposite direction and vernier divisions are marked in the opposite direction as that of the main scale.

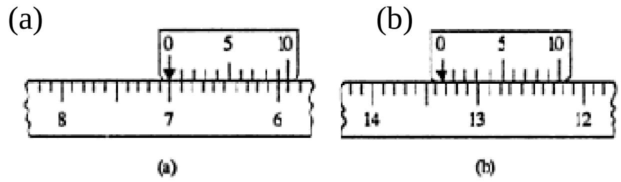

Figure 3. (a) Retrograde vernier, (b) measuring the length using a retrograde vernier scale (note the arrow)

In retrograde vernier scale, n vernier divisions are expanded into the space of (n + 1) main scale divisions, we say the vernier-scale ratio is [(n): (n + 1)]. Thus, in case of retrograde vernier, the divisions of the vernier scale will be larger than those on the main scale and will facilitate in easy reading.

Let d = value of smallest division on the main scale
v = value of smallest division on the vernier scale

Then, $v = \frac{n+1}{n} d$

Least count (LC) = $v - d = \frac{n+1}{n} d - d = \frac{d}{n}$

It is to be noted that the LC of direct and retrograde verniers are same.
Fig. 3(a) illustrates a retrograde vernier in which 11 parts of the main scale divisions coincide with 10 divisions of the vernier. The value of one smallest division on the main scale is 0.1 and the number of divisions on the vernier are 10. Therefore, the least count is $= \frac{0.1}{10} = 0.01$. The reading on the vernier in Fig. 3(b) is 13.34.

Extended vernier:
This type of vernier is similar to the direct vernier scale except that every second division is omitted. Therefore, in case of extended vernier scale, (2n − 1) divisions of the main scale are taken and they are divided into n equal parts.
Let d = value of smallest division on the main scale
v= value of smallest division on the vernier scale

Figure 4. (a) Extended vernier, (b) measuring the length using an extended vernier scale (note the arrow)

Then, $v = \frac{2n-1}{n} d$

Least count (LC) = $2d - v = 2d - \frac{2n-1}{n} d = \frac{d}{n}$

The extended vernier is, therefore, equivalent to a simple direct vernier in which every second graduation is engraved. The extended vernier is regularly employed in the astronomical sextant. Fig. 4(a) shows an extended vernier. It has 6 spaces on the vernier equal to 11 spaces of the main scale each of 1 mm. The least count is therefore $= \frac{1}{6}$ mm. The reading on the vernier illustrated in Fig. 4(b) is 2.67 mm.

Vernier to Circular scales:

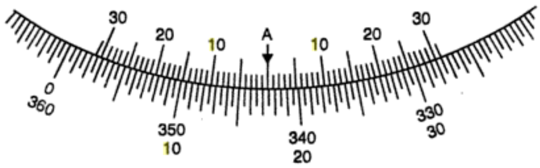

Figure 5. Vernier to circular scale

The above examples of verniers were for the linear scales. Verniers are also extensively used to circular scales in variety of scientific instruments. Fig. 5 shows typical arrangement of double *direct* verniers. In Fig. 5 the scale is graduated to 30´ and value of n = 30 on the vernier. Hence the least count is = d/n = 30´/30 = 1´. The clockwise angle reading (inner row) is 342^o 30´ + 05´ = 342^o 35´ and the counter clockwise reading (outer row) is 17^o 0´ + 26´ = 17^o 26´.

A Linear Scale and a Circular Scale

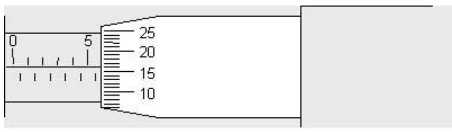

Figure 6. Linear scale and a circular scale

It has a linear scale called the main scale, and another scale called the **circular scale.** The **circular scale** can be rotated by a head screw. On turning the screw, the **circular scale** advances linearly on the main scale. The distance moved by the tip of screw when it is given one complete rotation, is called the **pitch** of the screw. Dividing the pitch of screw by the total number of divisions on the **circular scale**, we get the distance which the screw advances on rotating the screw by 1 division on its **circular scale.** This distance is called the least count (LC) of the instrument. Thus

$$LC = \frac{\text{Pitch}}{\text{Total number of division } (n) \text{ on the circular scale}}$$

Generally, the screw advances by 1 or ½ division on main scale when the screw is given one rotation. If there are total 100 division on its circular scale and the value of 1 division on main scale is 0.1 cm, then least count = 0.001 cm.

Net reading = linear scale reading + least count × circular scale reading.
In Fig. 6 the measurement corresponding to the given position will be 5.66 mm

Zero Error: When the studs are in touch, if the circular scale has crossed the reference line on the main scale by x divisions, the instrument is said to have a *negative zero error*. Therefore, the error is required to be added to the apparent reading. If the circular scale fails to reach the reference line on the main scale by x divisions, the instrument is said to have a *positive zero error*. Therefore, the error is required to be subtracted from the apparent reading.
The error is corrected by adding or subtracting the quantity, $y = x \times L.C.$

References:

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