General Instructions

A. General laboratory practices

a) Attendance is taken twice in each class (once when the student comes into the lab and once when the student leaves the lab.)

b) One experiment will be performed per day. Experiments will be performed by two students together. However, the readings have to be taken independently by each student. **The completed experiments have to be evaluated in the next class**. No credit will be given for more than 2 weeks delay after the experiment is performed. No viva will be taken during evaluation. Each experiment will be marked out of 10. Questions at the end of the experiment have to be pondered over. However, it is not compulsory to answer them, but the students who answer these questions will benefit.

c) Marks distribution:

Lab. work-book	: 25
Attendance	: 10
OTE (Objective Type Exam)	: 25
End sem. Viva	: 10
End sem. exam + viva during exam	: 25+5
Total	**: 100 Marks**

The details of all the exams will be displayed on the lab notice board before the exams.

If a student misses any of the above exams due to a *valid* reason, he/she has to get in touch with the section-in-charge immediately to know the possibility of a re-exam, if any. The student has to produce a leave letter, clearly stating the reason for missing the exam, signed by the faculty adviser for his/her request to be considered.

d) Only one repetition class will be available before the end-semester exam, which will be on the day of the end-semester viva.

e) Lasers must not be continuously SWITCHED-On and Off! Any breakage / non-functioning of instruments have to be brought to the notice of the teachers / lab technicians immediately.

f) While leaving the lab, the apparatus has to be arranged in a manner as it was initially provided to the student. Switch off the power to your experiment, when you leave the lab. Slippers have to be kept back in their place.

g) **Use of mobile phones in the class is not allowed**.

B. Performance of the Experiments

The students will do well to read the following guidelines before getting started with experiments:

(i) Read the chapters of the work-book on error analysis, measuring scales and on instruments that you will be handling during the lab classes.

(ii) *Study all the 'Background',* mentioned in the beginning of each experiment.

(iii) *Study the experiment* very carefully, and think of the steps you will take in carrying out the work. Read the experimental details (particularly the principle of the experiment which you need to perform on a day) from the work-book and reference books, before coming to the lab.

(iv) Check that you have all the required apparatus. Try to understand the working principles of all the components involved in the experimental set-up.

(v) Do the work cleanly and *methodically*, step by step.

(vi) Record all the data in your note book systematically in the Tables.

(vii) Read the same quantity several times to reduce *random errors*. Correct your data for *systematic errors*, where known.

(viii) The entire stress is given to the actual record made by the student in the Workbook that (s)he uses in the class. If a mistake is made in recording a value, pen through it and write the new value by its side. **Do *not overwrite***. ***Do not use pencil*** for recording the observations.

(ix) While performing the experiment, one of the observations has to be noted in front of the teacher and should be signed by the teacher.

(x) Do not perform any back-calculation.

(xi) Draw graphs, where necessary.

(xii) Put units in the final results.

(xiii) Compute the *proportional error*, and judge the number of significant figures to be kept in the result.

(xiv) Please keep in mind that the manuals, which have been provided to you, are just a basic guideline to perform the experiments. You are always encouraged to improve on your experiments beyond whatever mentioned in the manuals. Do not hesitate to ask your doubts.

C. Drawing graphs

If a change in the value of a quantity x causes a change in the value of another quantity y, we say that a *functional relationship* exists between x and y. If we use the functional relationship to determine y from a value of x, y is said to be the *dependent variable,* and x the *independent variable*. A visual display of the functional relationship between x and y is obtained by plotting them on a graph paper.

When the measured values of x and y are quite accurate and the functional relationship between them is known, it is easy to draw a graph by plotting the measured values. When the experimental data are not so accurate, one has to draw the 'best curve' that 'fits' the data.

A straight line represents the simplest functional relationship and is easy to draw. Even if the relationship between the quantities measured is not a straight line, mathematical manipulations may be made to obtain a straight line. Thus, if $y = ax^2$, or log y = A log x, where a and A are constant, we may plot y against x^2 or log y against log x, respectively, to obtain straight line graphs in the two cases. The best straight line that can be drawn from a given set of experimental data is determined by the *principle of least squares*.

In the analysis of many of the experiments in this lab one requires drawing straight-line graphs from a given set of data. There are two different ways of doing this: (i) draw it by eye-estimation (ii) draw it using the method of least squares.

Eye estimation

To draw a straight line graph using eye-estimation one follows a basic principle. One needs to ensure, by eye estimation that, apart from the data-points that lie on the graph you intend to draw, equal number of the other points must lie on either side of the line. Also, if one or two plotted points appear to deviate too much from a straight line then we have to reject these points. For example, if you have ten data points and you try to draw a line which has four of these points on it, then the remaining six must be equally distributed on either side of the line (three on each side). Of course, to have equal number of points on either side one always needs to have an even number of such data points. Hence, it becomes necessary to ensure that if you have an even number of data points then an even number subset of that data-set must lie on the proposed straight line. Similarly, if we have an odd number of data points then an odd number subset of data points must be made to lie on the line. Note that, for an even number of data points one can draw a best-fit line following the eye estimation method, for which none of the data points may actually lie on the line. With odd number of data, it is however necessary that at least one data-point lies on the line. It has to be admitted that the eye estimation method is subjective and not entirely accurate. However, it does give a good approximation if you are sufficiently careful while drawing your straight line graph.

Method of least squares

In this method one makes use of a more accurate mathematical method for drawing the best-fit straight line. As per this method, the straight line that gives the best fit to the data is the one for which the sum of the squares of the deviations in y from the straight line is a minimum. A rough *working rule* that is close to this criterion is the following: Find the arithmetic means $\bar{x}$ and $\bar{y}$ of all the x values and all the y values of the plotted points under question. Draw a straight line through the point $(\bar{x}, \bar{y})$ so that the plotted points lie on the line or are close to it such that the sum of the distances of the points above the line is nearly equal to that for the points below it. This straight line roughly gives the best fit to the data.

Let us briefly outline the method. Consider x and y as the quantities which are measured (with some instrument of a certain least count). x_n and y_n denote the measured values, N in number, i.e. n = 1, 2 ... N. We wish to draw a straight line graph which fits this data-set. Assume this straight line (which is to be drawn) to be given by the equation:

$$y = ax + b \qquad (1)$$

where, a and b are the coefficients which we wish to determine from the data-set.
Of course, to determine a and b we must have some principle. The principle used here is about minimising the error. How does one do that? One defines the error as:

$$E(a,b)=\sum_{n=1}^{N}(y_n-(ax_n+b))^2 \qquad (2)$$

Thereafter one obtains *a* and *b* by minimising *E*(*a*,*b*) using the relations:

$$\frac{\partial E}{\partial a}=0; \quad \frac{\partial E}{\partial b}=0 \qquad (3)$$

The above two conditions lead to the following simple equations for *a* and *b*

$$a\left(\sum_{n=1}^{N}x_n^2\right)+b\left(\sum_{n=1}^{N}x_n\right)=\sum_{n=1}^{N}x_ny_n \qquad (4)$$

$$a\left(\sum_{n=1}^{N}x_n\right)+bN=\sum_{n=1}^{N}y_n \qquad (5)$$

Solving the two simultaneous equations in *a* and *b*, we get

$$a=\frac{\left(\sum_{n=1}^{N}x_ny_n\right)-N\,\bar{x}\,\bar{y}}{\left(\sum_{n=1}^{N}x_n^2\right)-N\,\bar{x}^2} \qquad (6)$$

$$b=\frac{\bar{y}\left(\sum_{n=1}^{N}x_n^2\right)-\bar{x}\left(\sum_{n=1}^{N}x_ny_n\right)}{\left(\sum_{n=1}^{N}x_n^2\right)-N\,\bar{x}^2} \qquad (7)$$

where, $\bar{x}=\frac{\left(\sum_{n=1}^{N}x_n\right)}{N}$ and $\bar{y}=\frac{\left(\sum_{n=1}^{N}y_n\right)}{N}$.

In order to ensure that the above *a* and *b* do indeed give a minimum for *E*(*a*,*b*) one needs to verify (at the extremum) that

$$\frac{\partial^2E}{\partial a^2}>0; \qquad \frac{\partial^2E}{\partial b^2}>0; \qquad \Delta=\left(\frac{\partial^2E}{\partial a\partial b}\right)^2-\frac{\partial^2E}{\partial a^2}\frac{\partial^2E}{\partial b^2}<0 \qquad (8)$$

You can check that $\frac{\partial^2E}{\partial a^2}=2\sum_{n=1}^{N}x_n^2$; $\quad \frac{\partial^2E}{\partial b^2}=2N;\ \ and\ \ \Delta=-2N\left(\sum_{n=1}^{N}(x_n-\bar{x})^2\right)<0.$

Chi-square fitting

Chi-square (χ^2) fitting uses a measure of goodness of fit which is the sum of differences between observed (y_i) and expected ($a\,x_i+b$) outcome, each squared and divided by the square of the standard deviation of the errors associated with each measurement (σ_i^2).

$$\chi^2=\sum_i\left[\frac{y_i-(a\,x_i+b)}{\sigma_i}\right]^2 \qquad (9)$$

Thereafter, one obtains a and b by minimising $\chi^2(a,b)$ using the relations similar to equations (3)-(7). We can thus obtain:

$$a = \frac{\begin{vmatrix} \sum\frac{x_i y_i}{\sigma_i^2} & \sum\frac{x_i}{\sigma_i^2} \\ \sum\frac{y_i}{\sigma_i^2} & \sum\frac{1}{\sigma_i^2} \end{vmatrix}}{det(X)}, \qquad b = \frac{\begin{vmatrix} \sum\frac{x_i^2}{\sigma_i^2} & \sum\frac{x_i y_i}{\sigma_i^2} \\ \sum\frac{x_i}{\sigma_i^2} & \sum\frac{y_i}{\sigma_i^2} \end{vmatrix}}{det(X)} \tag{10}$$

where,
$$det(X) = \begin{vmatrix} \sum\frac{x_i^2}{\sigma_i^2} & \sum\frac{x_i}{\sigma_i^2} \\ \sum\frac{x_i}{\sigma_i^2} & \sum\frac{1}{\sigma_i^2} \end{vmatrix} \tag{11}$$

The errors associated with a and b can be obtained as:

$$\delta a = \frac{\sum\frac{1}{\sigma_i^2}}{det(X)} \quad \text{and,} \qquad \delta b = \frac{\sum\frac{x_i^2}{\sigma_i^2}}{det(X)} \tag{12}$$

Another quantity which can be calculated is the reduced χ^2 or χ^2 per degree of freedom (*DOF*). In this case, χ^2 can be evaluated using the values of a and b from equation (10), and divided by the degree of freedom (*DOF*).

DOF = Number of data points – Number of fitting parameters

We have a good fit if the reduced χ^2 is of the order of 1, and the fit is poor if this is much more than 1.

Exercise: You are given the following set of data, (x_n, y_n), where N=8.
(1.0, 2.3); (2.0, 4.1); (3.0, 6.5); (4.0, 8.0); (5.0, 10.3); (6.0, 11.9); (7.0, 14.6); (8.0, 16.4).
Draw the best fit line by:

(i) eye estimation
(ii) using the least squares fitting method (i.e., by finding a and b using Eqns. (6) and (7)).
(iii) using the chi-square method (i.e., by finding a and b using Eqns. (10)).

The ***general rules*** that should be followed in drawing graphs, straight lines or not, are the following:

(i) Draw bold lines on the graph paper to serve as x and y axes. The independent variable should be plotted along the x-axis, and the dependent variable along the y-axis. The input which can be varied independently and has a precise value is the independent variable. The measured variable for a given input is the dependent variable. Write the plotted quantity and its unit by the side of each axis.
(ii) Note the range of values to be plotted along the two axes. A small division along each axis is chosen to represent a convenient value of the quantity so that the available space on the graph paper is *well utilized* in

accommodating these ranges. If the straight line does not pass through the (*0,0*) point, then the origin can be shifted to any other value for a better plot.

(iii) At the large division marks along each axis, write the numerical values of the quantity to which they correspond.

(iv) Plot each pair of the variables and mark the point by a small dot surrounded by a small circle. It is unnecessary to write the coordinates of the point by its side. If you are plotting more than one set of data points in the same graph, then mark the data points of different sets with different symbols *ex*., circle, triangle, rectangle etc.

(v) Draw the best continuous smooth curve through the average of the points. Use a fine pencil for this purpose. The curve should normally pass through most of the plotted points; other points should be evenly distributed on the two sides of the curve. Points lying far away from the curve should be rejected.

(vi) When the graph is a straight line, use a scale to draw it. When the graph is not a straight line, take care not to introduce any sudden change of curvature. This may be checked by holding the graph horizontally at the eye level and looking tangentially.

(vii) It may be necessary to *read* a value from the graph. In that case, mark the corresponding point and draw its ordinate in broken lines.

Reference

1. William H. Press, Brian P. Flannery, Saul A. Teukolsky and William T. Vetterling, *Numerical Recipes in C: The Art of Scientific Computing*, Cambridge University Press (1992).