Date:

11. Compound Pendulum

Background

Compound pendulum
Simple harmonic oscillation
Radius of gyration

Aim of the experiment

i) To determine acceleration due to gravity, g, using a compound pendulum.
ii) To determine radius of gyration about an axis through the center of gravity for the compound pendulum.

Apparatus required

Compound pendulum
Stop watch

Theory

A rigid body which can swing in a vertical plane about some axis passing through it is called a compound or physical pendulum.

In Fig. 1, a body of irregular shape is pivoted about a horizontal frictionless axis through P and is displaced from its equilibrium position by an angle θ. In the equilibrium position the center of gravity G of the body is vertically below P. The distance GP is a and the mass of the body is m. The restoring torque for an angular displacement θ is

$$\tau = -mg\, a \sin\theta \qquad \ldots(1)$$

For small amplitudes,

$$I\frac{d^2\theta}{dt^2} = -\ mg\ a\ \theta \qquad \ldots(2)$$

where, I is the moment of inertia of the body through the axis P. Expression (2) represents a simple harmonic motion and hence the time period of oscillation is given by

$$T = 2\pi\sqrt{\frac{I}{mga}} \qquad \ldots(3)$$

Now, $I = I_G + ma^2$, where I_G is moment of inertia of the body about an axis parallel with axis of oscillation and passing through the center of gravity G.

$$I_G = m\,k^2 \qquad \ldots(4)$$

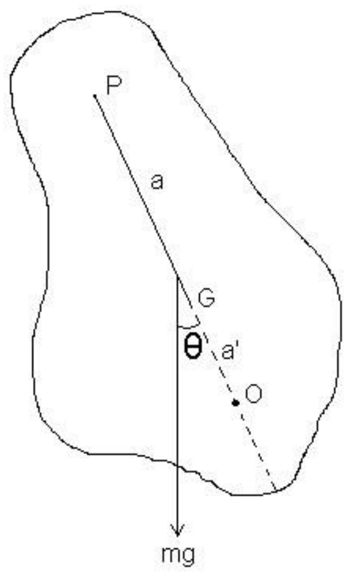


Fig. 1: Compound Pendulum

where, *k* is the radius of gyration about the axis passing through G. Thus,

$$T = 2\pi \sqrt{\frac{mk^2+ma^2}{mag}} = 2\pi \sqrt{\frac{\frac{k^2}{a}+a}{g}} \qquad \ldots(5)$$

Comparing expression (5) with an expression of time period $\left(T = 2\pi\sqrt{\frac{l}{g}}\right)$ for a simple pendulum suggests, $l = \frac{k^2}{a} + a$. This is the length of "equivalent simple pendulum". If all the mass of the body were concentrated at a point O, along PG produced such that $OP = \frac{k^2}{a} + a$, we would have a simple pendulum with the same time period. The point O is called the 'Centre of Oscillation'.

Now since

$$l = \frac{k^2}{a} + a$$

or,

$$a^2 - al + k^2 = 0 \qquad \ldots(6)$$

Equation (6) has two values of GP (or *a*), which produces the same length *l* as the length of the equivalent simple pendulum. Since one of the roots for equation (6) is a, the other root *a'* will satisfy

$$a + a' = l$$

and

$$aa' = k^2 \qquad \ldots(7)$$

Thus, if the body were supported on a parallel axis through the 'Centre of Oscillation' point O, if would oscillate with the same time period *T* as when supported at P. Now it is evident that there are an infinite number of points distant a and $\frac{k^2}{a} = a'$ from G in a rigid body. If the body were supported by an axis through G, the time period of oscillation would be infinite. From any other axis in the body the time period is given by expression (5).

The time period has a minimum value when $a + \frac{k^2}{a}$ is minimum, and that happen when *a* = *k*, and the corresponding time period is

$$T_{min} = 2\pi \sqrt{\frac{2k}{g}} \qquad \ldots(8)$$

This experiment can be performed with help of a rectangular metallic rod about 1 m long. This may be suspended on a knife-edge at various points along its length through circular holes drilled along the bore at about 2 or 3 cm intervals. (Fig. 2)

Procedure:

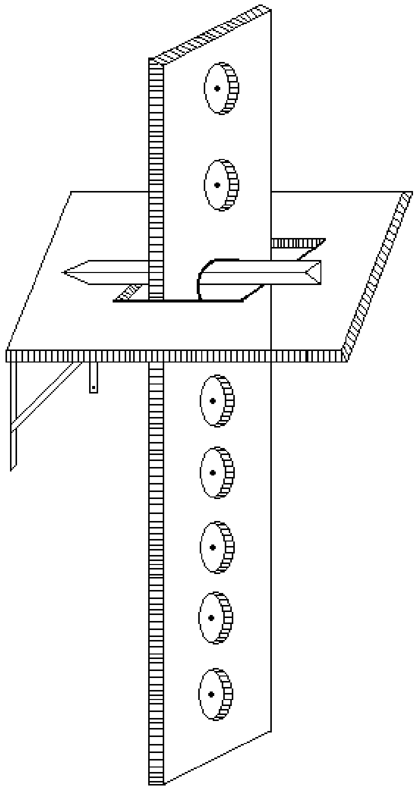


Fig. 2: Compound Pendulum

1) Level the knife-edge and suspend the bar at, say, every other hole in turn, and note time for twenty oscillations several times also note the distance of the hole from the center of the bar.

2) Having obtained a set of values for time periods T, and corresponding distances from the center of gravity, plot a curve with time period vs distance of suspension from the center of gravity. A curve such as shown in (Fig.3) will be obtained.

3) It will be found that curve is symmetrical about the line representing center of gravity. Draw any line CAGBD parallel to the axis. This cuts the curve in four points, which have the same time period. The equivalent length *l* for this time period is

$$l = \frac{AD+BC}{2} \quad \ldots(9)$$

The acceleration due to gravity is found using,

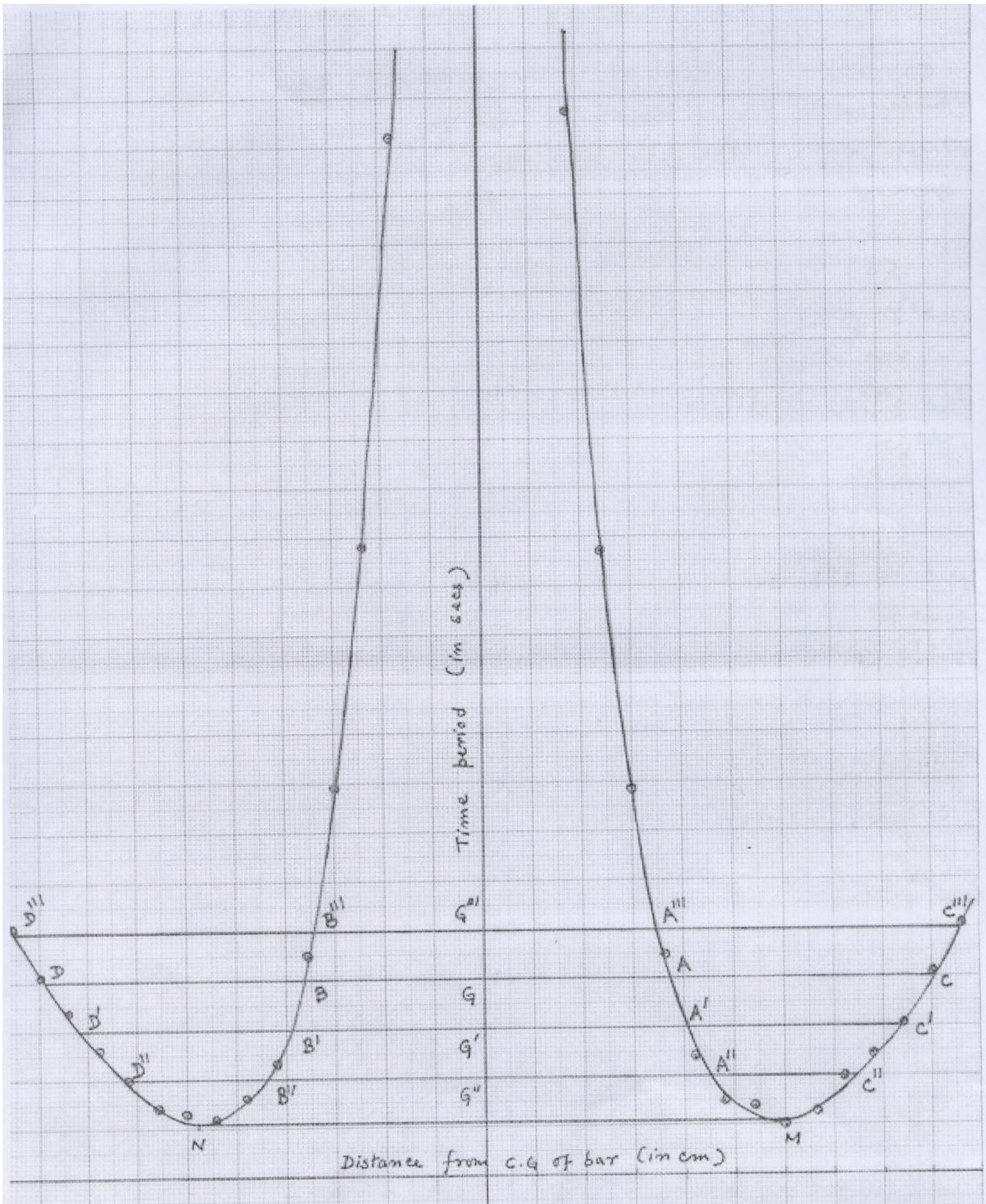
$$g = \frac{4\pi^2 l}{T^2} \quad \ldots(10)$$

4) Draw several lines parallel to CAGBD, (Fig.3) $C'A'G'B'D'$, $C''A''G''B''D''$ etc. and obtain the corresponding values of *l* and *T*. The mean value of $\frac{4\pi^2 l}{T^2}$ is used to calculate the value of *g*.

5) If now a tangent is drawn to the curve such as MN, then radius of gyration about an axis through center of gravity is equal to MN/2.

6) Further using equation (7) $k = \sqrt{aa'} = \sqrt{AG.GD}$. So, a second value of k may be found. The corresponding time period may be noted from the graph. Using equation (8) of may be evaluated.

7) Mean value of *k* may be obtained by averaging all values of *k* found from lines CAGBD, $C'A'G'B'D' \ldots$ etc, and the moment of inertia about a parallel axis through the center of gravity calculated using equation (4), where mass of the rod is obtained by direct weighing.


Time period (in secs)
Distance from C.G of bar (in cm)
D'''
D
D'
D''
N
B'''
B
B'
B''
G'''
G
G'
G''
A'''
A
A'
A''
C'''
C
C'
C''
M

(Fig 3)

One side of C.G.						Other side of C.G.					
Hole No.	**Distance from C.G. (in cm)**	**Time for 20 Oscillation (in s)**			**Time period** $T = \frac{t_1 + t_2 + t_3}{60}$ **(s)**	**Hole No.**	**Distance from C.G. (in cm)**	**Time for 20 Oscillation (in s)**			**Time period** $T' = \frac{t'_1 + t'_2 + t'_3}{60}$ **(sec)**
		t_1	t_2	t_3				t'_1	t'_2	t'_3	

Calculations & Results:

T_{min} =

S.N	Line	Time Period (in s)	l (in cm)	k (in cm)	k (in cm)	$g = \frac{4\pi^2 l}{T^2}$	Ave. g	$g = \frac{4\pi^2 (a+b)}{T^2_{min}}$	Ave. g
(i)	***CAGBD***	***T***	$\frac{AD+BC}{2}$	$\sqrt{AG \times GD}$ ***= (a)***	$\sqrt{BG \times GC}$ ***= (b)***	………	…….	………….	……
1									
2									
3									
4									
5									
6									

Acceleration due to gravity at Kharagpur is found to be equal to ………………. cm/s^2.

Radius of gyration about on axis through the center of gravity is found to be equal to (MN/2) … cm.

Error calculation

$$\frac{\delta g}{g} = \frac{\delta l}{l} + \frac{2\delta T}{T}$$

References

References: Worsenop & Flint : Advanced Practical Physics for Students
Resnick & Halliday : Physics

Graph: Compound Pendulum