Date:

4. Pohl’s Pendulum

Background

Damped oscillation
Forced oscillation
Logarithmic decrement
Creeping

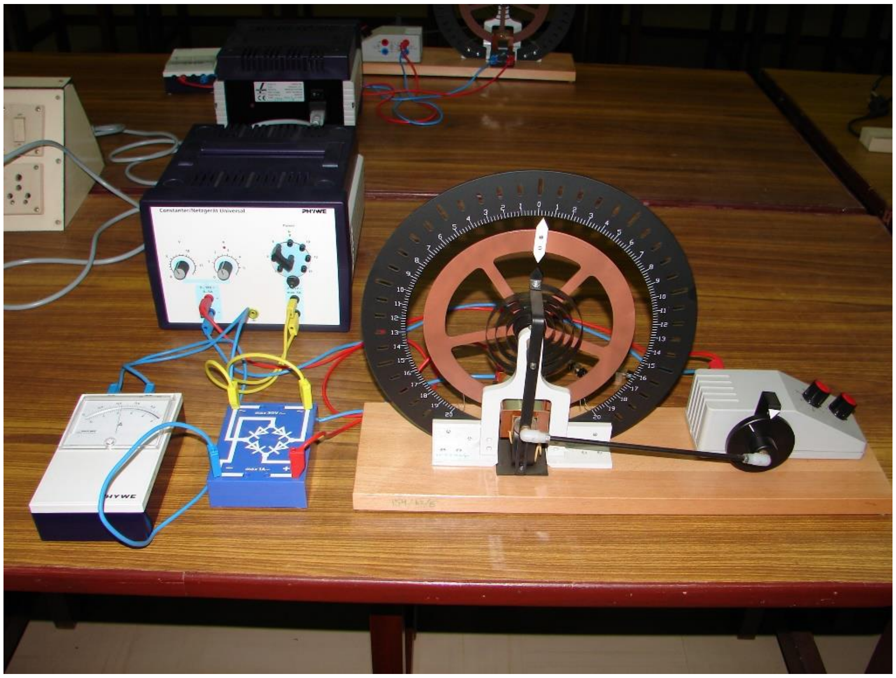
Aim of the experiment

1. Determination of damping constant of the pendulum for different eddy damping current.
2. Draw the resonance curve for the pendulum under different eddy damping current.
3. Estimation of the natural frequency of the pendulum.

Apparatus required

Torsion pendulum after Pohl
Power supply
Bridge rectifier
Stopwatch
Ammeter
Connecting cords

Pohl’s pendulum



Theory

In an oscillating system there is always a damping and hence the amplitude of the oscillator decreases with time. To maintain the amplitude an external forcing is required.
We shall study both these cases in the present experiment.

A. Underdamped oscillations:

In a torsional pendulum the restoring torque, M_1, and the damping torque (resistance), M_2, are given by,

$M_1 = -D\phi$ and $M_2 = -C\dot{\phi}$,

where ϕ = angle of rotation, D = torque per unit angle, C = factor of proportionality depending on the current which supplies the eddy current break. This results in the following equation of motion,

$$I\ddot{\phi} + C\dot{\phi} + D\phi = 0 \quad \text{...(1)}$$

where, I = pendulum's moment of inertia about the axis of rotation and $\ddot{\phi}$ = angular acceleration.
Dividing equation (1) by I we obtain,

$$\ddot{\phi} + 2\beta\dot{\phi} + \omega_0^2\phi = 0 \quad \text{...(2)}$$

where,

$$\beta = \frac{C}{2I} \quad \text{and} \quad \omega_0^2 = \frac{D}{I}.$$

β is called the "damping constant" and $\omega_0 = \sqrt{\frac{D}{I}}$ is the natural frequency of the undamped system.
If the pendulum is rotated to one side and released from rest at $t = 0$, such that $\phi = \bar{\phi}$ and $\dot{\phi}$ =0, at $t = 0$, then the solution of equation (2) is

$$\phi(t) = \phi_0 e^{-\beta t} \cos(\omega t + \delta) \quad \text{...(3)}$$

where, $\phi_0 = \bar{\phi}\sqrt{1 + \frac{\beta^2}{\omega^2}}$ and $\delta = \tan^{-1}\left(-\frac{\beta}{\omega}\right)$ with $\omega = \sqrt{\omega_0^2 - \beta^2} \geq 0$.

Solution (3) shows that the amplitude of the underdamped oscillations decreases exponentially with time.

The *logarithmic decrement* is defined as the natural logarithm of the ratio of successive amplitudes (see Fig 1.),

$$\lambda = \ln\frac{\phi_n}{\phi_{n+1}} = \beta T. \quad \text{...(4)}$$

where T= $2\pi/\omega$, is the time period of the underdamped oscillations.

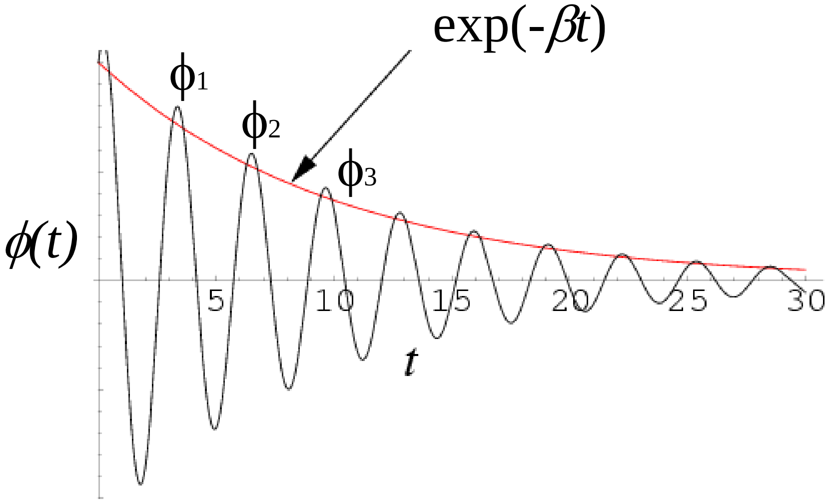


Fig. 1. Underdamped oscillations and successive amplitudes

For $\omega_0^2 = \beta^2$, the pendulum returns in a minimum of time to its initial position without oscillating (aperiodic case or critically damped). For $\omega_0^2 < \beta^2$, the pendulum returns asymptotically to its initial position (creeping or overdamped).

B. Forced oscillation

If the pendulum is acted on by an external torque $M_a = M_0 \cos \omega_a t$, then equation (2) changes to

$$\ddot{\phi} + 2\beta\dot{\phi} + \omega_0^2\phi = F_0 \cos\omega_a t \quad \ldots\ldots(5)$$

where, $F_0 = M_0/I$.

In steady state, the solution of this differential equation is

$$\phi(t) = \phi_a \cos(\omega_a t - \alpha) \quad \ldots\ldots(6)$$

where,

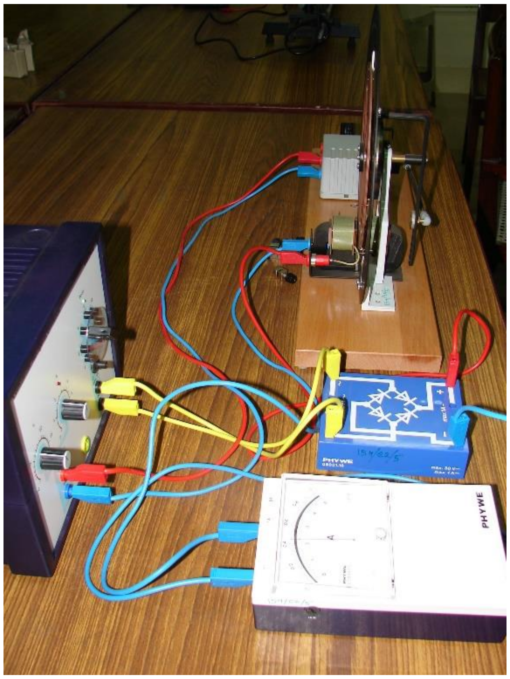
$$\phi_a = \frac{F_0}{\sqrt{(\omega_0^2 - \omega_a^2)^2 + 4\beta^2\omega_a^2}}. \quad \ldots\ldots(7)$$

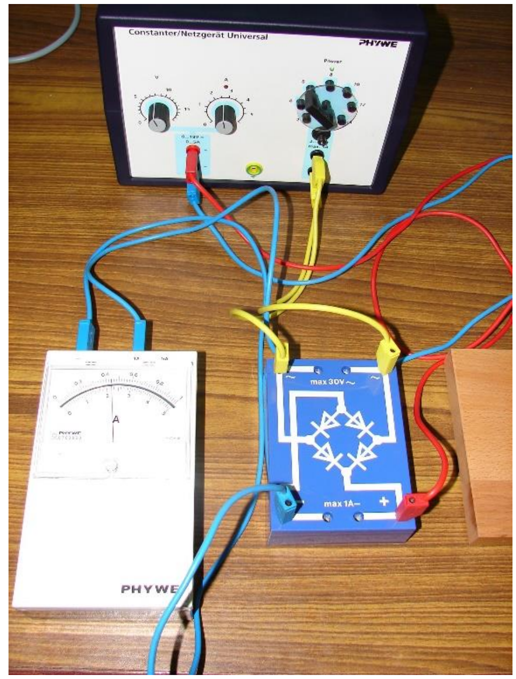
Furthermore, $$\tan\alpha = \frac{2\beta\omega_a}{\omega_0^2 - \omega_a^2} \ldots\ldots(8)$$

An analysis of equation (7) gives evidence of the following:

1. The greater F_0, the greater ϕ_a.
2. For a fixed value of F_0, amplitude ϕ_a exhibits a peak at a frequency, ω_{res}, given by
   $$\omega_{res} = \sqrt{\omega_0^2 - 2\beta^2} = \sqrt{\omega^2 - \beta^2} \ldots\ldots(9)$$
3. The greater β, the smaller ϕ_a.

Procedure





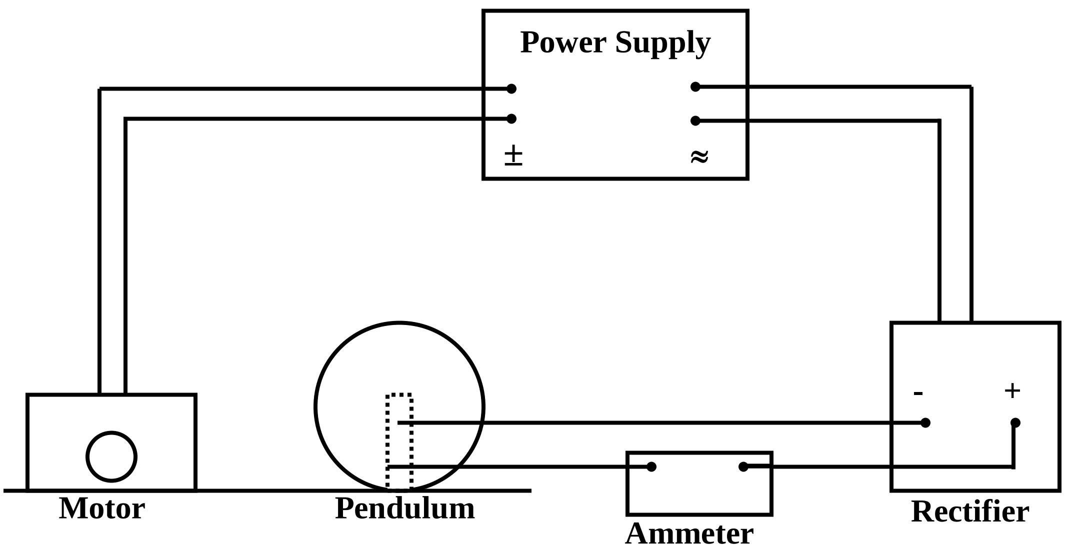


Fig. 2 Electrical connection for the experiment

A. Damped oscillations

1. Make the electrical connections as per diagram 2. Keep the motor switch off before switching on the power supply. Make sure that the pendulum pointer shows zero.

2. Find the period of oscillation, T, of the pendulum without damping current (natural damping). To minimize the error in measurement, deflect the pendulum completely to one side in the beginning and measure time for 20 oscillations repeatedly (say thrice).
3. Measure successive amplitudes (on the other side of initial deflection) of oscillations with time. Measure amplitudes at every time interval T, that is measure successive unidirectional amplitudes, $\phi_{0,}$ $\phi_{1,}$ $\phi_{2,}$ etc.
4. Repeat 3 giving the same initial deflection to minimize the errors in the measurement of amplitudes.
5. Repeat 2, 3 and 4 for different damping currents, i.e. by keeping the damping voltage knob say at 2*V*, 6*V*, 10*V* etc.
6. Successive unidirectional amplitudes as a function of time for different damping are plotted in a semi log graph paper (time on the normal axis and amplitude on the log axis). For a fixed damping plot should be a straight line. From the slopes of the plots, damping resistance, β, can be estimated, for various dampings. (Plot t *vs* $ln(\phi)$ to estimate damping constant for various damping currents.)

B. Forced oscillations

7. Keep the 'course' and 'fine' potentiometer knobs of the motor at low and mid values respectively. Keep the damping voltage at 2*V*. Adjust the voltage of the motor power supply (DC) to maximum.
8. The forcing frequency ω_a of the motor can be estimated by counting the number of turns eccentric disc is making per unit time using a stopwatch. Note the amplitude of the pendulum once it is stabilized. Slowly turn the 'coarse' to increase the forcing frequency. Again the stabilized amplitude is noted down. When the amplitude starts increasing, ω_a is changed in small steps using the 'fine' knob of the potentiometer to nearly locate the resonance amplitude (ϕres) and frequency (ωres). In each case forcing frequency is noted and the amplitude reading is taken after the pendulum stabilizes. Notice that in the steady state situation the forcing frequency and the pendulum frequency are same. For small values of damping care must be taken to choose the values of ω_a in such a way that the pendulum does not exceed its scale near the resonance.
9. Repeat step 8 for different damping values.
10. Plot amplitude versus forcing frequency graphs for various values of damping. Find the resonance frequency and amplitude from the graph and compare them with the estimates made earlier using the part A of the experiment.
11. Notice phase shifts between the forcing agent and the pendulum for frequencies far below the resonance frequency and also for frequencies far above it.

Observations

Least count the stop watch………………s

Table 1. Time period of oscillations with different damping currents

Damping current (in A)	No. of Oscillations	Time (in s)	Time period (in s)

Table 2. Maximum values of unidirectional amplitudes as a function of time for different damping

Damping current (A)	Time t (s)	Amplitude of oscillations (*a*)	*ln* (ϕ_a)	Time t (s)	Amplitude of oscillations (*b*)	*ln* (ϕ_b)
	0	ϕ_{a0} =		T/2 =	ϕ_{b0} =	
	T =	ϕ_{a1} =		3T/2 =	ϕ_{b1} =	
	2T =	ϕ_{a2} =		5T/2 =	ϕ_{b2} =	
	3T =	ϕ_{a3} =		7T/2 =	ϕ_{b3} =	

continued…

Damping current (A)	**Time t (s)**	**Amplitude of oscillations (*a*)**	***ln*** **(ϕ_a)**	**Time t (s)**	**Amplitude of oscillations (*b*)**	***ln*** **(ϕ_b)**
	0	ϕ_{a0} =		T/2 =	ϕ_{b0} =	
	T =	ϕ_{a1} =		3T/2 =	ϕ_{b1} =	
	2T =	ϕ_{a2} =		5T/2 =	ϕ_{b2} =	
	3T =	ϕ_{a3} =		7T/2 =	ϕ_{b3} =	

continued…

Damping current (A)	**Time t (s)**	**Amplitude of oscillations (*a*)**	***ln* (ϕ_a)**	**Time t (s)**	**Amplitude of oscillations (*b*)**	***ln* (ϕ_b)**
	0	ϕ_{a0} =		T/2 =	ϕ_{b0} =	
	T =	ϕ_{a1} =		3T/2 =	ϕ_{b1} =	
	2T =	ϕ_{a2} =		5T/2 =	ϕ_{b2} =	
	3T =	ϕ_{a3} =		7T/2 =	ϕ_{b3} =	
	0	ϕ_{a0} =		T/2 =	ϕ_{b0} =	
	T =	ϕ_{a1} =		3T/2 =	ϕ_{b1} =	
	2T =	ϕ_{a2} =		5T/2 =	ϕ_{b2} =	
	3T =	ϕ_{a3} =		7T/2 =	ϕ_{b3} =	

Table 3. Estimation of Damping constant and resistance for different damping currents

Damping current (A)	$\ln\left(\frac{\phi_{a(n)}}{\phi_{a(n+1)}}\right)$	$\ln\left(\frac{\phi_{b(n)}}{\phi_{b(n+1)}}\right)$	Ave. Logarithmic decrement λ	Time period T (sec) $=2\pi/\omega$	Damping constant $\beta=\lambda/T$	$\omega_{res} = \sqrt{\omega^2 - \beta^2}$

Table 4. Plot for frequency vs amplitude under forced oscillation (**Note**: T_1 is to be measured on the eccentric disc and, T_2 on the pendulum.)

Sl. No.	Damping current (A)	Time period of forcing oscillation in seconds			Freq. of oscillation (s^{-1})	Amplitude of oscillation			
		T_1	T_2	T_{av}		ϕ_1	ϕ_2	ϕ_3	ϕ_{av}

continued…

Sl. No.	Damping current (A)	Time period of forcing oscillation in seconds			Freq. of oscillation (s^{-1})	Amplitude of oscillation			
		T_1	T_2	T_{av}		ϕ_1	ϕ_2	ϕ_3	ϕ_{av}

continued…

Sl. No.	Damping current (A)	Time period of forcing oscillation in seconds			Freq. of oscillation (s^{-1})	Amplitude of oscillation			
		T_1	T_2	T_{av}		ϕ_1	ϕ_2	ϕ_3	ϕ_{av}

continued…

Sl. No.	Damping current (A)	Time period of forcing oscillation in seconds			Freq. of oscillation (s^{-1})	Amplitude of oscillation			
		T_1	T_2	T_{av}		ϕ_1	ϕ_2	ϕ_3	ϕ_{av}

Results:

Damping current (A)	Logarithmic decrement λ	Damping constant β (in Hz)	Resonance frequency ω_{res} (in Hz)		Resonance Amplitude ϕ_{res}	Natural freq. from damped osc. (in Hz) $\omega_0 = \sqrt{\omega^2 + \beta^2}$
			Observed	Estimated		

Precaution

Do not keep the pendulum in resonance condition without damping or with a very low damping for a long time

Error calculation:

Error in estimation of β

$\beta = [\ln \phi_n - \ln \phi_{n+m}]/mT$

$$\frac{\delta\beta}{\beta} = \frac{\delta T}{T} + \frac{1}{m\beta T}\left[\frac{\delta\phi}{\phi_n} + \frac{\delta\phi}{\phi_{n+m}}\right] \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(10)$$

Error in estimation of ω

$$\frac{\delta\omega}{\omega} = \frac{\delta T}{T}$$

Error in estimation of ω_0

$$\frac{\delta\omega_0}{\omega_0} = \frac{\omega\,\delta\omega + \beta\delta\beta}{\omega^2 + \beta^2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(11)$$

Questions:

1. What is eddy current?
2. Other than the eddy current damping force, what are the other forces those are affecting your experiment?
3. What do you expect if $\omega_0 = \beta$ or $\omega^2 < \beta^2$?
4. What are your conclusions about the phase relationship between the driver and the oscillator below and above resonance?
5. Give a simple example of forced oscillation.
6. What is the physical reason for the large amplitude oscillation at the resonance frequency?
7. Why the resonance curve broadens for higher damping?
8. Check equations 10 and 11.

Reference:

1. PHYWE LEP 1.3.27 Forced oscillations-Pohl's pendulum
2. H. J. Pain, The Physics vibrations and waves.

Graphs:

1. **Plot *t vs ln(ϕ)* to estimate damping constant for various damping currents.**

2. **Forced oscillation: Plot *forcing frequency vs amplitude* graphs for various values of damping.**